# Tracing of Cartesian Curves

by

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# Symmetry

1. The curve is symmetric about x-axis if the power of y occurring in the equation are all even, i.e.$f(x, -y) = f(x, y)$. e.g.$x = y^2$

2. The curve is symmetric about y-axis if the powers of x occurring in equation are all even, i.e.$f(-x, y) = f(x, y)$. e.g.$y = x^2$

# Continue...

3. The curve is symmetric about the line y= x, if on interchanging x and y, the equation remains unchanged, i.e. $f(y, x) = f(x, y)$.
   e.g.$x^2 = y^2$

# Continue...

3. The curve is symmetric about the line y= x, if on interchanging x and y, the equation remains unchanged, i.e. $f(y,x) = f(x,y)$.
   e.g.$x^2 = y^2$

4. The curve is symmetric in opposite quadrants or about origin if on replacing x by –x and y by –y, the equation remains unchanged,
   i.e.$f(-x,-y) = f(x,y)$.
   e.g.$x^2 = y^2$

# Origin

1. The curve passes through the origin if there is no constant term in the equation.

2. If curve passes through the origin, the tangents at the origin are obtained by equating the lowest degree term in $x$ and $y$ to zero.

3. If there are two or more tangents at the origin, it is called a node, a cusp or an isolated point if the tangents at this point are real and distinct, real and coincident or imaginary respectively

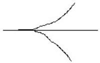
# Point of intersection

1. The point of intersection of curve with x and y axis are obtained by putting $y = 0$ and$x = 0$ respectively in the equation of the curve.

2. Tangent at the point of intersection is obtained by shifting the origin to this point and then equating the lowest degree term to zero.

# Special Points

1. **Cusps**: If tangents are real and coincident then the double point is called cusp.



2. **Nodes**: If the tangents are real and distinct then the double point is called node.

3. **Isolated Point**: If the tangents are imaginary then double point is called isolated point.

# Asymptotes

1. Asymptotes parallel to x-axis are obtained by equating the coefficient of highest degree term of $x$ in the equation to zero.

2. Asymptotes parallel to y-axis are obtained by equating the coefficient of highest degree term of y in the equation to zero.

# Region of Presence

1. This region is obtained by expressing one variable in terms of other, i.e., $y=f(x)$[or $x=f(y)$] and then finding the values of x (or y) at which y(or x) becomes imaginary. The curve does not exist in the region which lies between these values of x (or y).

# Trace the cissoid $y^2(2a - x) = x^3$

- **Symmetry:** The power of y in the equation of curve is even so the curve is symmetric about x-axis.
- **Origin:** The equation of curve dose not contain any constant term so the curve passes through the origin.

  To find tangents at the origin equating lowest degree term to zero,

$$2ay^2 = 0$$

$$=> y^2 = 0$$

$$=> y = 0$$

Thus x-axis be a tangent.

# Continue...

- **Points of intersection:** Putting $y = 0$, we get $x = 0$. Thus, the curve meets the coordinate axes only at the origin.
- **Asymptotes:**
  **a)** Since coefficient of highest power of $x$ is constant, there is no parallel asymptote to x- axis.
  **b)** Equating the coefficient of highest degree term of $y$ to zero, we get

$$2a - x = 0$$

$=> x = 2a$ is the asymptote parallel to y-axis.

# Continue...

- **Region:** We can write the equation of curve like $y^2 = \frac{x^3}{(2a-x)}$
- The value of $y$ becomes imaginary when $x < 0$ or $x > 2a$.
- Therefore, the curve exist in the region $0 < x < 2a$